Short Papers

A Low-Frequency Investigation into the Discontinuity Capacitance of a Coaxial Line Terminated in a Lossless, Dielectric-Loaded Circular Waveguide

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Abstract —**The problem of theoretically determining the normalized discontinuity capacitance of a dielectric-loaded coaxial termination has been reexamined and it is shown that, when frequency effects can be ignored, the solution to the problem may be expressed in the form of a rapidly convergent power series with power terms which depend only on the relative permittivity of the dielectrics and with coefficients which depend only on the line size. Values for the coefficients in the case of typical line sizes are presented and the accuracy of the power series solution is discussed.**

I. INTRODUCTION

The problem of accurately determining the discontinuity capacitance of a coaxial termination has received the attention of many authors, not least because of its importance both in standards work [1–3] and in permittivity studies of biological or other substances [4], [5]. In these respects, attention has in particular focused on the problem of theoretically determining capacitance changes due to permittivity effects when the frequency of operation is very low. It is the purpose of this communication to reconsider the problem in relation to the coaxial termination shown in Fig. 1 and to show how, in the electrostatic limit, one might infer the behavior of such changes as a function of changes in the permittivity. In order to do so, it will be necessary to examine the governing equations from a quasi-numerical standpoint. The analysis dealing with this particular problem is assumed to be well known and only the relevant results need therefore be considered.

II. FORMULATION

The circular waveguide region $0 \leqslant r \leqslant a$ is assumed to contain a loss-free dielectric of permittivity ϵ_2, and the coaxial waveguide region $b \leqslant r \leqslant a$ is assumed to contain a loss-free dielectric of permittivity ϵ_1. The electromagnetic boundary value problem which arises when the coaxial line is fed with the TEM mode may be addressed using either a variational formulation as described in [1] for the air-filled termination or an appropriate Green's function integral equation formulation allied to a moment-method solution. In both cases, it is possible to arrive at the same result, which relates the discontinuity capacitance of the termination to the solution of an infinite set of simultaneous equations whose coefficients are slowly convergent series of the Fourier–Bessel type. Specifically, it may be shown that the normalized discontinuity capacitance $\bar{C}$, which here is taken to mean the discontinuity capacitance normalized to the permittivity ratio ϵ_2/ϵ_1, is

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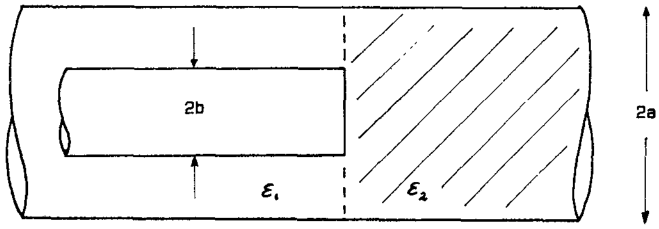

Fig. 1. Coaxial termination with extended outer conductor.

given by the expression

$$\bar{C} = \kappa\left(b_0 - \sum_{n=1}^{\infty} b_n x_n\right). \tag{1}$$

In the above, κ is a constant which is given by the equation

$$\kappa = 4\pi a \epsilon_1/(\ln(b/a))^2 \tag{2}$$

and the x_n are the solutions to the infinite set of simultaneous equations

$$\sum_{n=1}^{\infty} a_{mn} x_n = b_m \qquad (m = 1,2,\cdots). \tag{3}$$

The coefficients b_0, b_m and a_{mn} in these equations are the aforementioned series of the Fourier–Bessel type. They are given explicitly by the expressions

$$b_0 = \sum_{j=1}^{\infty} \frac{1}{\lambda_j^2 a^2 (\lambda_j^2 a^2 - k_2^2 a^2)^{1/2}} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 \tag{4}$$

$$b_m = \sum_{j=1}^{\infty} \frac{(-1)}{(\lambda_j^2 a^2 - k_2^2 a^2)^{1/2} (\lambda_j^2 a^2 - \mu_m^2 a^2)} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 \tag{5}$$

and

$$a_{mn} = \sum_{j=1}^{\infty} \frac{\lambda_j^2 a^2}{(\lambda_j^2 a^2 - k_2^2 a^2)^{1/2} (\lambda_j^2 a^2 - \mu_m^2 a^2)(\lambda_j^2 a^2 - \mu_n^2 a^2)} \cdot \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 + \frac{\epsilon_1}{\epsilon_2} \frac{\delta_{mn}}{4(\mu_m^2 a^2 - k_1^2 a^2)^{1/2}} \cdot \left[\frac{J_0^2(\mu_m b)}{J_0^2(\mu_m a)} - 1\right]. \tag{6}$$

In the above, the usual Bessel function notation has been employed and the $\lambda_j a$ $(j = 1,2,\cdots)$ are the ordered zeros of the Bessel function $J_0(\lambda_j a)$. The $\mu_m a$ $(m = 1,2,\cdots)$ are the ordered zeros of the mixed Bessel function $J_0(\mu_m a) Y_0(\mu_m b) - J_0(\mu_m b) Y_0(\mu_m a)$ and δ_{mn} denotes the Kronecker delta symbol; k_1 and k_2 denote the different wavenumbers in the respective dielectrics and the expressions are valid in the wavelength range $\lambda_j a > k_2 a$. In the limiting case when $\epsilon_1 = \epsilon_2$, these equations are in essence the same as those obtained by Risley [1], although the latter's results were presented in a somewhat more complicated form. It is a straightforward matter to compute the normalized discontinuity capacitance both for various line sizes and for

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different permittivities, and some typical low-frequency results for the air-filled termination are shown in Table I. The results shown in this table are suitable for standards work, being in good agreement with results which have been predicted elsewhere using other methods [2], [3]. The other results are also included in the table for the purpose of making an immediate comparison.

III. Electrostatic Solution

In the case of the electrostatic limit, that is, when the wavenumbers k_1 and k_2 are equated to zero, it is possible to employ readily established results from the theory of Fourier–Bessel series [6] to reshape the coefficients b_m and a_{mn}, thereby providing a fresh insight into both their behavior and the behavior of the solution to the system of equations. Specifically, the result (A1) of the Appendix may be employed so that b_m may be rewritten in the form

$$b_m = \frac{1}{\mu_m a} \sum_{j=1}^{\infty} \frac{1}{\lambda_j a(\lambda_j a + \mu_m a)} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 \tag{7}$$

and the result (A2) together with (A1) may be similarly employed to present a_{mn} in the form

$$a_{mn} = \frac{(-1)}{(\mu_m a + \mu_n a)} \sum_{j=1}^{\infty} \frac{1}{(\lambda_j a + \mu_m a)(\lambda_j a + \mu_n a)} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 + \frac{(1+\epsilon_1/\epsilon_2)\delta_{mn}}{4\mu_m a}\left[\frac{J_0^2(\mu_m b)}{J_0^2(\mu_m a)} - 1\right]. \tag{8}$$

From (4), b_0 is quite simply given by the expression

$$b_0 = \sum_{j=1}^{\infty} \frac{1}{\lambda_j^3 a^3}\left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2. \tag{9}$$

The principal advantage to be gained by reshaping the coefficients in this manner is that the opportunity arises to determine the x_n, and hence the normalized discontinuity capacitance $\overline{C}$, as a power series expansion in $(1+\epsilon_1/\epsilon_2)^{-1}$ with coefficients which depend only on the line size. Specifically, an iterative scheme which involves a trivial matrix inversion is employed to obtain solutions to the governing matrix equation (3). In order to establish the scheme, the coefficient matrix of (8) is first decomposed into the sum of two matrices. One of these matrices contains elements which are simply the infinite series terms of (8) and the other matrix is purely diagonal, being formed from the remaining terms. The decomposed matrix equation is then premultiplied by the inverse of the diagonal matrix, and the subsequent expression is rearranged to establish the scheme. Additional details are quite straightforward and the matter need not be entertained further. As a result, the normalized discontinuity capacitance can be realized in the form

$$\overline{C} = \kappa \sum_{p=0}^{\infty} \gamma_p (1+\epsilon_1/\epsilon_2)^{-p}; \quad \gamma_0 = b_0. \tag{10}$$

Values of the leading coefficients in this expansion have been computed in the case of the aforesaid terminations and the results are shown in Table II. Clearly, the coefficients suffer appreciable decay and the power series appears to converge rapidly for all values of the permittivity ratio. The corresponding values for the normalized discontinuity capacitance are quickly obtained for all permittivity ratios using (10) in conjunction with the data in Table II. In the case of the air-filled termination, there is very good agreement with the low-frequency results depicted in Table I.

TABLE I
Normalized Discontinuity Capacitance for the Air-Filled Termination

TERMINATION (air filled ϵ_r =1.000635)	NORMALIZED DISCONTINUITY CAPACITANCE (fF) FREQUENCY IN Hz	
	10^3	10^9
24.3 Ohm (2a=19.05mm)	399.02 ref. [2] 398.88 398.80 ref. [3]	400.18 ref. [2] 400.08 ------
50.0 Ohm (2a=19.05mm)	217.40 ref. [2] 217.05 217.00 ref. [3]	218.07 ref. [2] 217.72 217.70 ref. [3]
50.0 Ohm (2a=14.2875mm)	163.04 ref. [2] 162.80 162.70 ref. [3]	163.336 ref. [2] 163.09 163.00 ref. [3]
50.0 Ohm (2a=14.0mm)	159.76 ref. [2] 159.53 159.40 ref. [3]	160.039 ref. [2] 159.79 ------
50.0 Ohm (2a=7.0mm)	79.88 ref. [2] 79.76 79.70 ref. [3]	79.917 ref. [2] 79.80 79.70 ref. [3]

TABLE II
Values of the Coefficients in the Power Series Expansion

n	COEFFICIENTS γ_n LINE SIZE IN OHMS 24.3 (a/b=1.5)	50.0 (a/b=2.30291)
0	0.064390	0.152950
1	-0.004508	-0.018542
2	-0.001020	-0.004295
3	-0.000300	-0.001263
4	-0.000097	-0.000403
5	-0.000032	-0.000132
6	-0.000011	-0.000044
7	-0.000004	-0.000015
8	-0.000001	-0.000005
9	-0.000000	-0.000002

Although the results which have been obtained using (10) are strictly valid only in the limiting case of zero frequency, there will inevitably be a small range of frequencies over which the results may still be of practical significance. For example, it is apparent from the text that in the case of the air-filled termination, the low-frequency formula is accurate to within about 0.3 percent for frequencies up to 1 GHz. The extent of the frequency tradeoff against permittivity which is required to sustain confidence in the low-frequency formula may be ascertained by comparing results

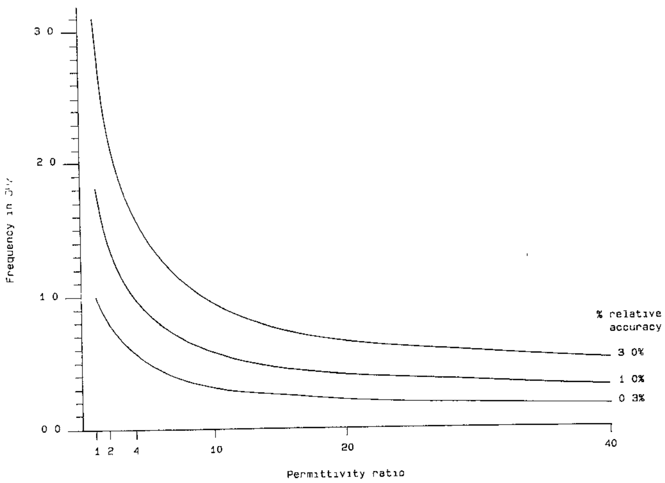

Fig 2 Relative accuracy bounds for the low-frequency formula.

in the frequency-dependent case with those from the formula. The results of a comparison in the case of the 24.3-Ω line are shown in Fig. 2 and similar results in respect of the 50.0-Ω line of the same outer diameter were also obtained, but they were found to be no less representative than those shown in the figure. For any point on any one of the curves in this figure, there corresponds a permittivity ratio and a frequency beyond which the specified relative accuracy cannot be sustained.

As a matter of interest, the results contained herein were obtained using a desktop micromputer, and all computations involving Bessel functions were carried out using the polynomial approximation representation for such functions [7]. The numbers of modes used to match the field in the annular domain to that in the circular domain were 50 and 100, respectively, and although no attempt has been made to determine the absolute accuracy of the computations, the results compare favourably with those of [2] in the case of the air-filled termination, for which an accuracy of about ±0.1 fF was reported.

Appendix

It may be deduced from an established result in the theory of Fourier–Bessel series that, for the $\lambda_j a$ and $\mu_m a$ as defined in the text, the following formulas are satisfied:

$$\sum_{j=1}^{\infty} \frac{1}{\left(\mu_m^2 a^2 - \lambda_j^2 a^2\right)} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 = 0 \qquad \text{(A1)}$$

and

$$\sum_{j=1}^{\infty} \frac{1}{\left(\mu_m^2 a^2 - \lambda_j^2 a^2\right)^2} \left[\frac{J_0(\lambda_j b)}{J_1(\lambda_j a)}\right]^2 = \frac{1}{4\mu_m^2 a^2}\left[\frac{J_0^2(\mu_m b)}{J_0^2(\mu_m a)} - 1\right]. \qquad \text{(A2)}$$

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A Printed Circuit Stub Tuner for Microwave Integrated Circuits

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***Abstract*—A novel microwave tuning element capable of continuous adjustment has been realized in the form of a planar printed circuit. As such, it is suitable for incorporation into microwave integrated circuits (MIC's), where it can be used for fine-tuning the impedance match between two parts of a circuit when the two parts are either subject to variations due to manufacturing tolerances or are difficult to model. In either case, the tuner is a compact on-circuit tuning facility which does not have to be removed after use. The tuner has been shown to have unique impedance-transforming properties, being capable of matching any realizable impedance to a 50-Ω load. Its main part consists of a coupled-line section and across the gap in the section at different places along its length are positioned two short-circuit bridge conductors. Movement of these bridges produces the variation in impedance transformation.** 3-10/8612437

I. Introduction

Stub tuners in coaxial form have been widely used for many years. Coaxial tuners usually comprise two or three short-circuit stubs of adjustable length connected in shunt with a main "through" line with some appropriate separation. Their chief application has been in the field of microwave measurements where there is a need for continuous adjustment of the impedance presented to a device under test in order to optimize some other parameter such as power transfer or noise figure. Notable examples include load pull measurements on large signal amplifying devices and noise parameter measurements on low-noise FET's.

While being well suited to the measurement role, coaxial tuners or tuners with the same basic circuit concept cannot be included as integral parts of microwave integrated circuits (MIC's). There are certain types of circuit where inclusion of a continuously variable tuning element would be extremely useful. They are mostly narrow-band circuits where perhaps a large spread in device characteristics results in a need for individual circuit tuning or where the use of a nonlinear device has resulted in

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